The Quarter-Wave Transformer Prototype Circuit*

LEO YOUNG†, SENIOR MEMBER, IRE

Summary—A quarter-wave transformer not only changes impedance levels, but will also behave as a band-pass filter. In practice, however, band-pass filters are usually required to terminate in equal input and output impedances. They often consist of several direct-coupled cavities, which are similar to transformers whose impedance steps have been replaced by reactive obstacles.

It is demonstrated how one can synthesize a quarter-wave transformer, and then "distort" it to obtain a direct coupled cavity filter with a predictable performance. This is illustrated and confirmed by numerical examples.

The method is particularly convenient to use in reverse. The quarter-wave transformer prototype is easily derived for a direct-coupled cavity filter which has already been designed by another approximate method, and thus gives an independent evaluation of its performance. If necessary, the filter can then be redesigned, as illustrated in this paper.

Introduction

THIS PAPER is concerned with a novel method of design of direct-coupled cavity filters. Previous methods have relied on lumped constant equivalent or prototype circuits, or at least have made use of the concept of Q as it occurs in those circuits. A survey of these methods has recently been given by Riblet.[1]

Instead of starting with an L-C circuit, another transmission line circuit is here taken as the prototype. This is a quarter-wave transformer, or a half-wave filter, which can be synthesized exactly.[2] It might be expected that by taking one transmission line circuit as a model or prototype for another transmission line circuit, it should be possible to relate their performances more accurately than when the final transmission line filter is based on a lumped-constant equivalent circuit. That this can indeed be so will be demonstrated by numerical examples in this paper.

The Homogeneous Quarter-Wave Transformer

The homogeneous ideal quarter-wave transformer is described by an impedance function $Z(p)$, where[2]

$$p = -j \cot \theta, \quad (1)$$

θ being the electrical length of each section of the multi-section cascaded transformer (Fig. 1).

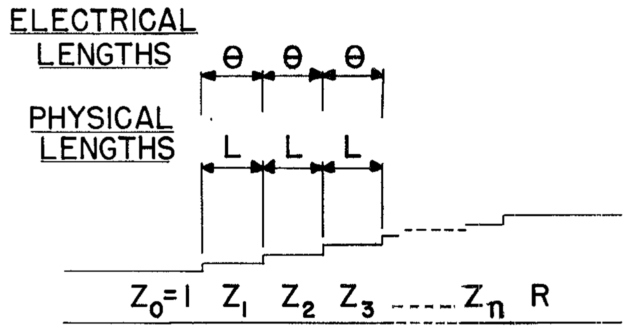

Fig. 1—Quarter-wave transformer ($\theta=\pi/2$ and $L=\frac{1}{4}\lambda_g$ at center frequency).

The insertion loss polynomial, P_L, is given by

$$P_L = 1 + P_n(\mu^2), \quad (2)$$

where P_n must be positive for real values of $\mu = \cos\theta$. If all roots of P_n are to occur at real frequencies, they must all be double roots for P_n to stay positive. Hence P_L is of the form

$$P_L = 1 + Q_n^2(\mu), \quad (3)$$

where Q_n is even or odd in μ. Riblet showed further that, given an insertion loss polynomial of the form (3), one can always synthesize a quarter-wave transformer having this same P_L. Following Collin, $Q_n(\mu)$ may be set equal to $kT_n(\mu/\mu_0)$ for Tchebycheff performance,[3] or $k\mu^n$ for maximally flat (Butterworth) performance.

The author[4] has tabulated the values of the characteristic impedance ratios $Z_1, Z_2, Z_3, \cdots$, (Fig. 1) for the homogeneous quarter-wave transformer, to give an equal ripple (Tchebycheff) response. The length L of each section (Fig. 1) is a quarter guide wavelength at "center frequency" and is defined by

$$L = \frac{\lambda_{g1}\lambda_{g2}}{2(\lambda_{g1}+\lambda_{g2})}, \quad (4)$$

where λ_{g1} is the greatest and λ_{g2} is the smallest guide wavelength in the pass band.

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† Stanford Res. Inst., Menlo Park, Calif. Formerly with Electronics Div., Westinghouse Electric Corp., Baltimore, Md.

[1] H. J. Riblet, "A unified discussion of high-Q waveguide filter design theory," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 359–368; October, 1958.

[2] H. J. Riblet "General synthesis of quarter-wave transformers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 36–43; January, 1957. The half-wave filter is an extension of the quarter-wave transformer concept, which will be described in this paper.

[3] R. E. Collin, "Theory and design of wide-band multisection quarter-wave transformers," Proc. IRE, vol. 43, pp. 179–185; February, 1955. Our scale factor μ_0 is Collin's p and Riblet's s. T_n is the Tchebycheff polynomial of degree n, and k^2 is a constant, called the pass-band tolerance.

[4] L. Young, "Tables for cascaded homogeneous quarter-wave transformers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 233–237, April, 1959; and vol. MTT-8, pp. 243–244, March, 1960.

The bandwidth W is defined by

$$W = 2\left(\frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g1} + \lambda_{g2}}\right). \tag{5}$$

Given the output-to-input impedance ratio R (taken greater than unity) and the bandwidth W, the relative characteristic impedances are tabulated in Young's article[4] for values of R from 1 to 100, and W from 0 (maximally flat) to 120 per cent.

Since the quarter-wave transformer converts between two impedance levels with the minimum reflection inside a specified pass band, it behaves as a bandpass filter. From the filter point of view, it is unfortunate that the input and output impedances are different. The next section shows how a band-pass filter with equal, or nearly equal, input and output impedances can be achieved.

Direct Synthesis of the Homogeneous Half-Wave Filter

To avoid the monotone increase in impedance levels of the "Tchebycheff" quarter-wave transformer, the theory may be extended to a cascade of lines in which each section is 180 electrical degrees long at center frequency. Instead of a quarter-wave transformer, there results a half-wave filter, which is a direct-coupled cavity-resonator type of filter,[5] with the resonant cavities defined by the characteristic impedance discontinuities (Fig. 2). The synthesis may be carried out directly, or may be based on the design of a quarter-wave transformer prototype. The latter approach is sketched in below.

Synthesis Based on the Quarter-Wave Transformer

Synthesis by Quarter-Wave Transformer Tables[4]

To find the appropriate conversions, regard every other step in the half-wave filter as an equivalent circuit consisting of one 90° and one (−90°) line length, with a step in between, as illustrated in Fig. 3. They are equivalent in the sense that they provide identical normalized impedances with respect to the respective characteristic impedances. This follows from the identity

$$\frac{1}{T}\begin{pmatrix}1 & \Gamma\\ \Gamma & 1\end{pmatrix} \equiv \begin{pmatrix}\pm j & 0\\ 0 & \mp j\end{pmatrix}\frac{1}{T}\begin{pmatrix}1 & -\Gamma\\ -\Gamma & 1\end{pmatrix}\begin{pmatrix}\mp j & 0\\ 0 & \pm j\end{pmatrix}. \tag{5a}$$

Note particularly that the 90° lines in the equivalent circuit are perfectly frequency independent. When this substitution is made for every other step in the half-wave filter (Fig. 2), it contracts into a quarter-wave transformer with one difference, that each line section is now twice as frequency sensitive as a quarter-wave section of line. To use the tables[4] for designing a half-wave filter, the procedure in a typical case might be as follows:

1) The maximum permissible VSWR in the given pass band and the number of sections (cavities) is specified.
2) From the "Maximum VSWR" tables, find the parameter R by looking under the column for *twice* the bandwidth actually specified.
3) With R so determined, and still using twice the bandwidth specified, find from the tables the impedance ratios $Z_1, Z_2, Z_3, \cdots$.
4) Invert every other impedance step, making successive line impedances either

$$Z_1,\ \frac{Z_1^2}{Z_2},\ \frac{Z_1^2 Z_3}{Z_2^2},\ \frac{Z_1^2 Z_3^2}{Z_2^2 Z_4},\ \frac{Z_1^2 Z_3^2 Z_5}{Z_2^2 Z_4^2}, \tag{6}$$

or

$$\frac{1}{Z_1},\ \frac{Z_2}{Z_1^2},\ \frac{Z_2^2}{Z_1^2 Z_3},\ \frac{Z_2^2 Z_4}{Z_1^2 Z_3^2},\ \frac{Z_2^2 Z_4^2}{Z_1^2 Z_3^2 Z_5}, \tag{7}$$

instead of Z_1, Z_2, Z_3, Z_4, Z_5 (if $n=4$, $Z_5=R$; if $n=3$, $Z_4=R$; etc.).

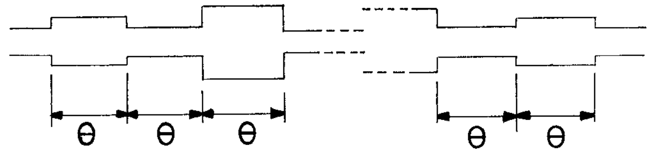

Fig. 2—Half-wave filter. θ=electrical length ($\theta=\pi$ at center frequency).

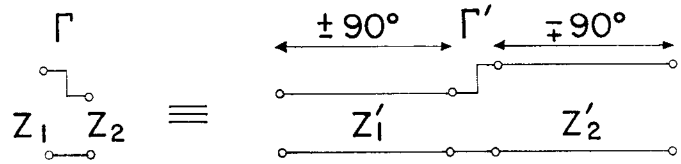

Fig. 3—Equivalent circuit of an impedance step. Impedance ratio: $Z_2'/Z_1'=Z_1/Z_2$. Reflection coefficient: $\Gamma'=-\Gamma$.

General Synthesis Procedure Based on the Quarter-Wave Transformer

For $n>4$, or $R>100$, or half-wave filter bandwidths in excess of 60 per cent, no tables are as yet available. In this case, μ_0 is first determined from the half-wave filter bandwidth W by

$$2W = 2\left(\frac{\text{maximum frequency} - \text{minimum frequency}}{\text{arithmetic mean frequency}}\right) = \frac{4}{\pi}\sin^{-1}\mu_0. \tag{8}$$

Note that $2W$, not W, has to be used for the half-wave filter. (For dispersive guides, replace frequency by reciprocal guide wavelength.)

[5] S. B. Cohn, "Direct-coupled-resonator filters," Proc. IRE, vol. 45, pp. 187–196; February, 1957.

In place of step 2), R is determined from the formula[3]

$$\frac{(R+1)^2}{4R} = 1 + k^2 T_n^2\left(\frac{1}{\mu_0}\right), \tag{9}$$

where

$$k^2 = P_m - 1 = \frac{|\Gamma_m|^2}{1 - |\Gamma_m|^2} = \frac{(V_m - 1)^2}{4V_m}, \tag{10}$$

P_m, $|\Gamma_m|$, and V_m being the maximum values of the insertion loss function, the reflection coefficient, and the VSWR, respectively.

Instead of referring to the tables,[4] Riblet's method is now used, with μ_0 from (8) and R from (9), *i.e.*, as if a quarter-wave transformer of twice the desired bandwidth were being designed.

Finally every other impedance step is again inverted.

Condition for Equal Input and Output Impedances

We observe from (6) or (7) that if $n=1$ or 3, the input and output impedances are the same, since[2,3] $Z_i Z_{n+i-1} = R$; whereas for $n=2$ or 4, they are not equal. This result can be generalized for odd and even n, respectively.

Lemma: The output-to-input impedance ratio of the half-wave filter is equal to the VSWR of the filter at center frequency.

Corollary: The input and output impedances of a half-wave filter are equal if and only if the filter is matched at center frequency.

Proof of Lemma: At center frequency, each line length is 180° by definition of the half-wave filter. Now input impedances are unchanged when adding or subtracting 180° line sections. Therefore, at center frequency, all intermediate line sections may be removed, leaving the input and output lines directly connected. This proves both the lemma and its corollary.

It is clear from the lemma that all Tchebycheff half-wave filters consisting of an odd number of sections will have equal input and output impedances, since $T_n(0) = 0$ if n is odd. Similarly, all Tchebycheff half-wave filters consisting of an even number of sections cannot have equal input and output impedances, since $T_n(0) \neq 0$ if n is even, except in the limiting case when Tchebycheff becomes maximally flat performance.

Numerical Results

To obtain numerical confirmation of these results, a nondispersive two-section quarter-wave transformer was designed and analyzed, and the half-wave filter derived from it was also analyzed. The two response curves so obtained are shown superposed in Fig. 4.

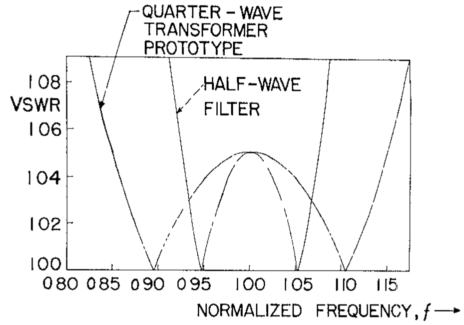

Fig. 4—VSWR against frequency of half-wave filter and its quarter-wave transformer prototype.

The prototype quarter-wave transformer chosen has $n=2$, $R=5$ and a bandwidth of 30 per cent. From the tables,

$$Z_0 = 1,\quad Z_1 = 1.5142,\quad Z_2 = 3.30208,\quad Z_3 = R = 5. \tag{11}$$

The normalized frequency response plotted in Fig. 4 confirms the bandwidth of 30 per cent and gives a maximum VSWR of 1.051 as predicted in Table I of Young.[4]

For the half-wave filter, (6) yields:

$$Z_0 = 1,\quad Z_1 = 1.5142,\quad Z_2 = 0.6944,\quad Z_3 = 1.05125. \tag{12}$$

This is also plotted in Fig. 4 and, as expected, gives a 15 per cent bandwidth (just half that of the quarter-wave transformer), but the same maximum VSWR of 1.051, which is also the output-to-input impedance ratio of the half-wave filter.

Direct Coupled Cavity Filters

The homogeneous quarter-wave transformer and half-wave filter perform like a transmission line loaded at equal intervals by obstacles of real and constant reflection coefficients. They resemble the direct-coupled cavity resonator filter,[5] in which each cavity is a section of uniform transmission line bounded by reactive obstacles.

The existence of an exact synthesis procedure for the ideal quarter-wave transformer and half-wave filter suggests using these circuits as "prototypes" for the approximate design of direct-coupled cavity filters.[6] This is analogous to the use of lumped constant "equivalent"

[6] The derivation of the half-wave filter from the quarter-wave transformer was itself a similar (but exact) procedure. The present application calls not merely for the inversion of impedance steps (having real Γ), but their replacement by reactive obstacles (complex Γ).

circuits, for which synthesis procedures and formulas have been worked out by Darlington[7] and others. Cohn,[5] for instance, has thus worked out some very useful formulas. The author can attest to the accuracy of Cohn's formulas for most cases of practical interest. Nevertheless, there are times when they must be expected to, and do, break down. This happens, for instance, when the cavity length is no longer close to 180 electrical degrees, as is stipulated in Fig. 14 of Cohn's article.[5]

The Quarter-Wave Transformer Prototype

Any transmission line loaded at intervals (Fig. 5) can be represented by a transformer circuit consisting of line lengths θ_1, θ_2, θ_3, etc., separated by impedance step transformers (Fig. 6). The reflection-coefficient of the first step, Γ_1, should be equal to the reflection coefficient of the first obstacle, and so on for Γ_2, Γ_3, etc. Each Γ is real but is no longer constant, varying in amplitude with frequency. Similarly the θ's of the prototype transformer (Fig. 6) are no longer equal to each other, or even commensurable, and may all vary differently with frequency.

Each obstacle in Fig. 5 has two symmetrically located reference planes giving it a real reflection coefficient. At center frequency, each reference plane is separated from the next one by an electrical length of 90 degrees.[8] The separations between obstacles at center frequency are $\phi_1, \phi_2, \cdots$ electrical degrees, as shown. It follows that this filter is represented by a half-wave filter equivalent as shown in Fig. 6, with $\theta_1 = \theta_2 = \cdots = 90°$ at center frequency. It would have a frequency behavior like that of a quarter-wave transformer if the Γ's remained constant, and the θ's remained equal to each other.

Now, if the Γ's increased or decreased slowly together with frequency, then a performance still not very different from a quarter-wave transformer's might be expected. Also, the θ's increase or decrease together at about the same rate for similar obstacles (*e.g.*, shunt inductances). We shall therefore average their rate of change, compare it to that of an originally 90° long line, and use it to predict the filter bandwidth.

The reference planes, which in Fig. 5 are shown 90° apart at center frequency, shift with frequency and determine the changes in the θ's. Their movement can be traced to two main causes:

1) Treating the transfer matrix of each obstacle as constant, the electrical separation between adjacent reference planes at any frequency is equal to the *change* in the initial electrical separation ϕ between the corresponding obstacles.
2) Any frequency dependence of the obstacle parameters may cause the reference planes to move.

[7] S. Darlington, "Synthesis of reactance four-poles which produce prescribed insertion loss characteristics," *J. Math. and Phys.*, vol. 18, pp. 257–353; September, 1939.
[8] *I.e.*, the filter is "synchronously tuned."

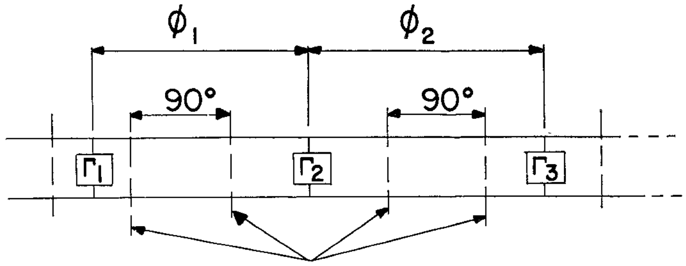

Fig. 5—Direct-coupled cavity filter (synchronously tuned). Reference planes with real Γ are 90° apart at center frequency. All parameter values ($\Gamma_1, \Gamma_2, \cdots, \phi_1, \phi_2, \cdots$) pertain to center frequency.

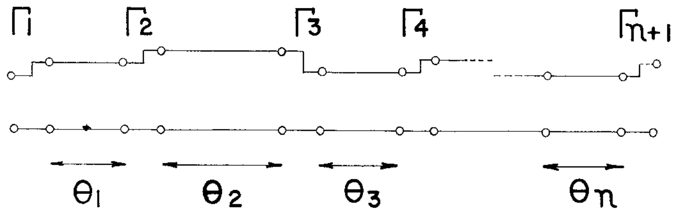

Fig. 6—Equivalent circuit of cascade of two ports.

Thus, if each obstacle is a shunt inductance, its susceptance is inversely proportional to frequency, and the reference plane positions are functions of the susceptance values.

Consider, for instance, the three-cavity symmetrical filter shown in Fig. 7, which uses shunt susceptances b_1, b_2, b_2, b_1. At center frequency, the spacings are

$$\phi_1 = 90° + \tfrac{1}{2}\{\arctan(b_1/2) + \arctan(b_2/2)\}$$
$$\phi_2 = 90° + \arctan(b_2/2), \tag{13}$$

in order to satisfy the synchronous tuning condition which requires that adjacent reference planes with real Γ be 90° apart at center frequency. If the b's were constant, the incremental separations of the reference planes over a bandwidth W due to "Cause 1)" would be

$$\phi_1 W; \ \phi_2 W; \ \text{and} \ \phi_1 W. \tag{14}$$

The change in magnitude of each b causes additional separations over the band W due to "Cause 2)" of

$$\frac{1}{2}\left\{\arctan\left(\frac{b_1}{2-W}\right) - \arctan\left(\frac{b_1}{2+W}\right) + \arctan\left(\frac{b_2}{2-W}\right) - \arctan\left(\frac{b_2}{2+W}\right)\right\},$$

$$\arctan\left(\frac{b_2}{2-W}\right) - \arctan\left(\frac{b_2}{2+W}\right), \ \text{and}$$

$$\frac{1}{2}\left\{\arctan\left(\frac{b_2}{2-W}\right) - \arctan\left(\frac{b_2}{2+W}\right) + \arctan\left(\frac{b_1}{2-W}\right) - \arctan\left(\frac{b_1}{2+W}\right)\right\}, \tag{15}$$

respectively.

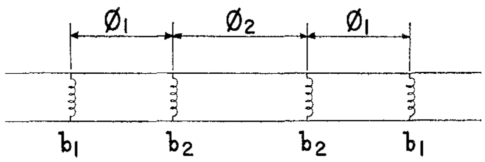

Fig. 7—Direct-coupled cavity filter using shunt inductances. ϕ_1, ϕ_2 are spacings at center frequency; b_1, b_2 are susceptances at center frequency.

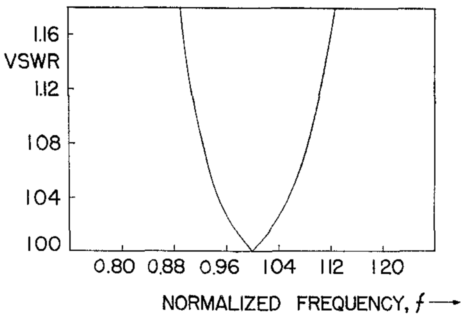

Fig. 8—VSWR against frequency of direct-coupled three-cavity filter designed by *L-C* equivalent circuit (example 1).

Together, (14) and (15) give the total movement over the band W. When they are averaged and divided into $(90W)^\circ$, they also show how much the original quarter-wave transformer bandwidth has contracted. This may be used to predict the filter bandwidth.

This procedure can be applied to any number of sections or cavities, and to couplings other than shunt inductive elements, provided their frequency behavior is known.

Numerical examples will illustrate the working of this method. Excellent agreement was obtained between predicted and subsequently analyzed performance. The method of solution is essentially a trial-and-error procedure, in which a quarter-wave transformer has to be found which, after "distortion," meets the filter requirements.

Numerical Examples

Specification and Description

A symmetrical direct-coupled cavity filter using inductive coupling posts is to be synthesized in coaxial line, to the following specifications:

number of cavities (n)	3,
maximum VSWR	1.1 approximately,
bandwidth (W)	0.3 (= 30 per cent).

The filter structure is shown schematically in Fig. 7. At center frequency, the shunt susceptances are b_1 and b_2, their reflection coefficients are Γ_1 and Γ_2, and the spacings are ϕ_1 and ϕ_2, as shown.

Two design methods are now presented, one by a lumped constant equivalent circuit, and one using the quarter-wave transformer prototype.

Design by L-C Equivalent Circuit

Example 1: Cohn's design formulas[5] for $n=3$, $W=30$ per cent, and maximum VSWR = 1.143, yield

$$b_1 = 0.42849, \qquad b_2 = 1.29085. \tag{16}$$

This corresponds to

$$\Gamma_1 = \frac{b_1}{\sqrt{4+b_1^2}} = 0.20949,\ \Gamma_2 = \frac{b_2}{\sqrt{4+b_2^2}} = 0.54229, \tag{17}$$

and makes, by (13),

$$\phi_1 = 112.465^\circ, \qquad \phi_2 = 122.837^\circ, \tag{18}$$

which is appreciably less than the near 180° spacings demanded by the approximation.[5] When this filter was analyzed, the frequency response curve shown in Fig. 8 was obtained. Instead of a Tchebycheff response with three zeros and two "hoops," there is only one zero and no hoop. To understand this failure of the *L-C* equivalent circuit method, consider the quarter-wave transformer prototype corresponding to (17). Taking $Z_0=1$, this transformer has

$$\begin{aligned} Z_1 &= \frac{1+\Gamma_1}{1-\Gamma_1} &&= 1.4981, \\ Z_2 &= Z_1\left(\frac{1+\Gamma_2}{1-\Gamma_2}\right) &&= 5.1657, \\ Z_3 &= R/Z_1 &&= 17.8120, \\ R &= Z_2^2 &&= 26.6844. \end{aligned} \tag{19}$$

From the tables,[4] we see that no transformer with $R=26.6844$ has a Z_1 less than 1.5295, which value occurs only for the maximally flat transformer. Therefore, we would indeed expect a frequency response like that obtained in Fig. 8.

Design by Quarter-Wave Transformer Prototype

Example 2: For our first trial solution, we select a prototype transformer with $R=25$ and (transformer) bandwidth of 45 per cent, for which the maximum VSWR is 1.06. From the tables[4] (with $Z_0=1$), one obtains by interpolation

$$\begin{aligned} Z_1 &= 1.5955, \\ Z_2 &= 5.0000, \\ Z_3 &= 15.670, \\ R &= 25.000, \end{aligned} \tag{20}$$

yielding

$$\Gamma_1 = \frac{Z_1-1}{Z_1+1} = 0.22943, \quad \Gamma_2 = \frac{Z_2-Z_1}{Z_2+Z_1} = 0.51619, \tag{21}$$

and hence

$$\begin{aligned} b_1 &= \frac{2\Gamma_1}{\sqrt{1-\Gamma_1^2}} = 0.47143, \\ b_2 &= \frac{2\Gamma_2}{\sqrt{1-\Gamma_2^2}} = 1.2054. \end{aligned} \tag{22}$$

Also

$$\arctan(b_1/2) = 13.263°, \quad \arctan(b_2/2) = 31.077°, \tag{23}$$

and therefore

$$\phi_1 = 112.170°, \quad \phi_2 = 121.077°. \tag{24}$$

We now wish to predict the filter bandwidth W. The average ϕ is

$$\frac{\phi_1 + \phi_2 + \phi_1}{3} = 115.139°. \tag{25}$$

If the susceptances did not change with frequency and remained equal to their initial values b_1, b_2, then the bandwidth would be reduced to $(90/115.139)\times 45$ per cent$=35.2$ per cent. The effect of the variation of the susceptances over the band is given by (15) and reduces this further. To calculate the quantities in (15), we must guess a value for W. Taking this as 0.3, (15) becomes

$$\tfrac{1}{2}\{15.500° - 11.583° + 35.355° - 27.658°\} = 5.807°,$$

$$35.355° - 27.658° = 7.697°, \tag{26}$$

and again

$$5.807°,$$

which have an average value of 6.437°. Since this is the incremental separation due to the change in the susceptances in a 30 per cent change in frequency about the center, the 6.437° increase is equivalent to an increase in the average line length of

$$6.437° \div 0.3 = 21.46°. \tag{27}$$

Moreover, it is clear that, although (27) was derived by assuming a 30 per cent bandwidth, it is a measure of the average rate of movement of the reference planes, and this average will not be rapidly affected by the exact length of the frequency interval used for averaging. Hence, the exact guessed value of W to obtain (27) is not critical.

We therefore anticipate, by (25) and (27), a reduction in bandwidth from the 45 per cent of the quarter-wave transformer to

$$\frac{90}{115.14 + 21.46} \times 45 \text{ per cent} = 29.7 \text{ per cent}. \tag{28}$$

The filter given by (22) and (23) was analyzed and its frequency response is given in Fig. 9. Its maximum VSWR is 1.03, and its bandwidth (for 1.03 VSWR) is 24 per cent, instead of 1.06 and 29.7 per cent, respectively, predicted from the quarter-wave transformer prototype of (20).

Example 3: It is seen from Fig. 9 that the bandwidth is still too small. To increase the bandwidth, one sees that from the transformer tables[4] b_1 should be increased and b_2 decreased. We try

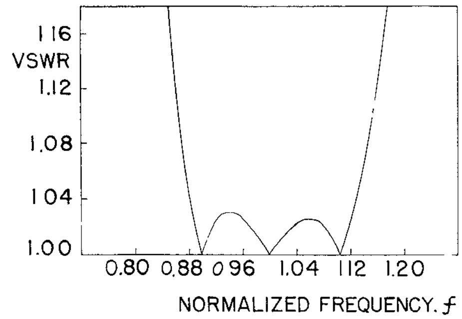

Fig. 9—VSWR against frequency of direct-coupled three-cavity filter designed from quarter-wave transformer prototype (example 2).

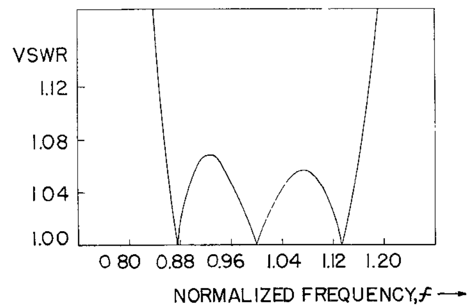

Fig. 10—VSWR against frequency of direct-coupled three-cavity filter designed from quarter-wave transformer prototype (example 3).

$$b_1 = 0.50000, \quad b_2 = 1.2000. \tag{29}$$

This gives

$$\phi_1 = 112.500°, \quad \phi_2 = 120.964°, \tag{30}$$

and

$$\Gamma_1 = 0.24254, \quad \Gamma_2 = 0.51449, \tag{31}$$

so that the transformer prototype (with $Z_0 = 1$) is

$$\begin{aligned} Z_1 &= 1.6404, \\ Z_2 &= 5.1159, \\ Z_3 &= 15.955, \\ R &= 26.172, \end{aligned} \tag{32}$$

which the tables show has a maximum VSWR of 1.10 and a (transformer) bandwidth of 52.8 per cent. The filter bandwidth W is again predicted as in the last example, and is expected to be 34.6 per cent. This filter was also analyzed and its frequency response is shown in Fig. 10. It is seen that the maximum VSWR is 1.07 instead of 1.10 as predicted; the bandwidth (for 1.07 VSWR) is 30.6 per cent instead of 34.6 per cent, as predicted (for 1.10 VSWR).

Concluding Remarks

It has been demonstrated that one can start with a quarter-wave transformer, which is a microwave circuit that can be synthesized to a given specification, and "distort" it to obtain a direct coupled cavity filter with a predictable performance. Some examples were analyzed numerically, and the predicted performance was closely confirmed.

This method is usually tedious to synthesize a filter *ab initio*, but it is quite easy to use in reverse, *i.e.* to derive the quarter-wave transformer prototype from a direct coupled cavity filter which has already been designed by another method. This was found to lead to a quick and accurate evaluation of its performance. If this predicted performance turns out to be inadequate, the filter can then be redesigned as illustrated in this paper.

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Broad-Band Ridge Waveguide Ferrite Devices*

E. S. GRIMES, JR.†, MEMBER, IRE, D. D. BARTHOLOMEW†, D. C. SCOTT†, MEMBER, IRE, AND S. C. SLOAN†, MEMBER, IRE

Summary—**The design and development of a medium CW power level, 1.57:1 bandwidth, quadruply-ridged circular waveguide Faraday rotator and two high CW power, 2:1 bandwidth, double ridge rectangular waveguide isolators are discussed.**

The rotator is designed in quadruply-ridged circular waveguide with a ferrite configuration somewhat different from that proposed by other investigators. It can be made to exhibit broadband rotation and large rotation per unit length of ferrite section, and may be used in most medium CW power level applications. Forty-five degree rotation is achieved over the 7.0-kMc to 11.0-kMc band.

The isolators operate from 2.0 kMc to 4.0 kMc in DR-37 waveguide and from 3.8 kMc to 7.6 kMc in D-34 waveguide respectively. The reverse to forward wave attenuation ratio exceeds 10.0 db to 1.0 db for both isolators.

INTRODUCTION

THE development of microwave components with operating bandwidths in excess of previously established maximums has been necessitated by the requirements of modern microwave systems. Perhaps the most widely used method for maintaining good performance characteristics of a microwave component over a large operating bandwidth is to reduce the variation of guide wavelength with frequency. The guide wavelength for any air-filled microwave transmission line is determined from the formula

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda/\lambda_c)^2}},$$

where

λ = operating wavelength
λ_g = guide wavelength
λ_c = cutoff wavelength.

Thus, for operating frequencies far above the cutoff frequency the variation in guide wavelength with frequency is greatly reduced. This condition can be brought about in circular and rectangular waveguides by using ridges protruding into the guide, thus lowering the dominant mode cutoff frequency without appreciably affecting the next higher mode cutoff frequency.[1,2] There, the resultant increase in bandwidth of the transmission line allows operation far above cutoff.

It is the purpose of this paper to present design information on a quadruply-ridged circular waveguide Faraday rotator and two double-ridge rectangular waveguide isolators.

FARADAY ROTATOR

A number of techniques have been proposed for maintaining rotation constant with frequency. In terms of maximum bandwidth the most successful ones known to the authors have utilized quadruply-ridged wave-

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† Sperry Microwave Electronics Co., Div. of Sperry Rand Co., Clearwater, Fla.

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